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Setting inventory levels of CONWIP flow lines via linear programming

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Abstract
This paper treats the problem of setting the inventory level and optimizing the buffer allocation of closed-loop flow lines operating under the constant-work-in-process (CONWIP) protocol. We solve a very large but simple linear program that models an entire simulation run of a closed-loop flow line in discrete time to determine a production rate estimate of the system. This approach introduced in Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011) for open flow lines with limited buffer capacities is extended to closed-loop CONWIP flow lines. Via this method, both the CONWIP level and the buffer allocation can be optimized simultaneously. The first part of a numerical study deals with the accuracy of the method. In the second part, we focus on the relationship between the CONWIP inventory level and the short-term profit. The accuracy of the method turns out to be best for such configurations that maximize production rate and/or short-term profit.

Keywords: CONWIP, flow lines, inventory, linear programming

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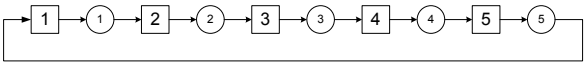
1 Flow lines with stochastic processing times under CONWIP control

A flow line with **CON**stant **W**ork **I**n **P**rocess (CONWIP) is characterized by a constant number of workpieces (the CONWIP level) circulating in the line. This constant number can be due to a fixed number of pallets or production authorization cards (PACs), see Buzacott and Shanthikumar (1993: 490).

Figure 1: Example of a CONWIP system with 5 stations



Such a CONWIP system is specified by $k = 1, ..., K$ serially arranged work stations $M_1, \ldots, M_K$, each followed by a corresponding (downstream) buffer of size b_k. See as an example in Figure 1 a system with $K = 5$ stations depicted as squares and buffers represented by circles. It is assumed that in front of the first station an unlimited amount of raw material is available. Each workpiece on a machine or in a buffer is attached to a pallet so that the total number of workpieces in the system is constant and equals the CONWIP level. When finished workpieces reach the buffer behind the last machine M_K, they are unloaded from the pallets. Then new workpieces are immediately loaded on the pallets to be next processed at the first machine M_1. If the stations are arranged in a circle or as a U-shape (which is often the case in practice), then the unloading/loading can be combined and included into the model with a single load/unload station. The transportation times between the work stations as well as the times for (un)loading the pallets are negligible and are assumed to be zero.
In this paper, we assume random effective processing times at each station. The effective processing time of a workpiece consists of the time to process the workpiece, which can be random, e.g., due to manual operations, and possibly also random

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repair times if the station is unreliable, e.g., due to (random) machine failures. This can lead to blocking and starving and a production rate of the line below the capacity of the bottleneck station if it operates in isolation. It is both economically important and scientifically challenging to quantify the impact of local buffer sizes and the global CONWIP inventory level on the production rate of the line.

Good surveys about methods for the analysis of flow lines are found in Dallery and Gershwin (1992) and Li, Blumenfeld, Huang, and Alden (2009). Recent literature surveys of closed-loop systems are given in Gershwin and Werner (2007) and Resano and Luis Pérez (2008). There is a wide range of applications of closed-loop flow lines in manufacturing. For example, Resano and Luis Pérez (2008) analyzed real automobile assembly lines and preassembly lines as a network of closed loops. Li, Blumenfeld, Huang, and Alden (2009) gave examples of different applications of closed-loop lines with a constant number of carriers for the automotive industry. Hopp and Roof (1998) reviewed different methods of setting the work-in-process (WIP) level in pull systems and analyzed a dynamic control of the WIP level to reach a target production rate within a given bound on the cycle time. Onvural and Perros (1989) approximated the throughput of a CONWIP system numerically and present methods to optimize the CONWIP level.

In general, three approaches for the analysis of stochastic flow lines have been widely established: exact probabilistic analysis, decomposition methods, and discrete-event simulation (DES). Both exact methods and (approximate) decomposition approaches are typically very fast, but also inflexible as they rely on quite specific assumptions about the stochastic behavior of the production system. DES, on the other hand, is extremely flexible, but often requires a substantial computational effort to evaluate a single configuration precisely. Neither method can be easily combined with the powerful optimization methodology of linear programming, see Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011). Our objective in this paper is therefore to close this gap for the particular case of CONWIP flow lines.

The basic idea of our approach originally introduced in Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011) is to approximate the stochastic behavior of a discrete-material flow line operating in continuous time within a large discrete-time linear program (LP). An attractive feature of this approach is the possibility to combine simulation and optimization within a single linear optimization framework. Previous LP-based models of stochastic flow lines were formulated in continuous time, see Abdul-Kader (2006), Johri (1987), Matta and Chefson (2005), and Schruben (2000). Due to the continuous time modeling approach, they could not easily model buffer sizes as decision variables, which is possible in our approach and important in the context of flow line optimization.

The main contribution of this paper is twofold: First, we extend the linear programming approach for open flow lines to analyze closed loops. This results in a very flexible approach for the performance analysis of stochastic CONWIP systems. It can be applied to simultaneously optimize the CONWIP level *and* the buffer allocation to maximize the average production rate or short-term profit. Note that for a *given* buffer allocation, it may also be possible to quickly determine an optimal CONWIP level via a limited number of DES. However, as soon as the buffer allocation has to be optimized as well, the search space explodes and an enumeration based on DES becomes impractical. This difficulty is overcome via our approach. To the best of our knowledge, it is the first 1-step approach for the design of CONWIP systems that prevents time-consuming DES of several configurations of the flow line. The second contribution of our paper is the practically very important result that the accuracy of our method is actually best for those CONWIP levels that maximize production rate and/or profit! This is a non-obvious result that we find very appealing from the application point of view.

The remainder of the paper is organized as follows. In Section 2, a discrete-time linear program is developed to evaluate CONWIP systems with a given CONWIP level and either finite or infinite buffer capacities. This evaluation model is extended to an optimization model, where both the CONWIP level and the buffer spaces are decision variables. In both models, the objective is to maximize the respective production rate estimate. Another extension deals with an economic objective function, which is based on gross margins and holding cost per product unit. The numerical studies in Section 3 present results on the accuracy of the method as well as results for the economic optimization of

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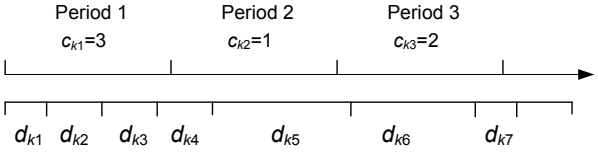
CONWIP lines. In Section 4, we summarize the most important findings of the paper and give an outlook on further research topics.

2 Linear programming modeling of CONWIP flow lines

2.1 Outline of the approach

In our approach the behavior of a discrete-material flow line operating in continuous time is approximated by a linear program (LP) that includes a discrete-time dynamic production-inventory model with continuous production quantities. The number of workpieces that can be processed at a work station of the line during a period (i.e., the production capacity) results from a simulation run in continuous time. In this simulation run we assume that the work station operates in isolation so that it can neither be blocked nor starved. The realizations of the stochastic processing times are transferred via sampling to realizations of maximum production capacities c_{kt} for each work station k and period t. If one considers a sequence of simulated processing times or durations d_{kw} to process an ordered set of workpieces w at a work station k, one just has to count the number of finished workpieces within period t. An example is shown in Figure 2, see Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011). In the upper part of Figure 2, three discrete-time periods 1 to 3 are depicted. In the lower part, the durations of seven consecutively processed workpieces are shown. Three workpieces are finished during period 1, one during period 2, and two during period 3. This procedure yields the capacity of the considered work station c_{kt} for the periods 1 to 3 as the realization of a stochastic count process.

Figure 2: Sampling of discrete-time processing rates



Two conditions must hold so that this discrete-time modeling approach based on production capacities c_{kt} can provide a reasonable picture of the production process in continuous time. Firstly, the sampling frequency must be high enough, i.e., the time periods must be short enough to yield a reasonable representation of the individual processing times. Secondly, the simulation run must be long enough and represent processing of enough workpieces to get a stochastically valid picture of the randomness of the processing times. That means that a substantial number of periods is required within the linear program, each with a specific sampled production capacity c_{kt}. For a detailed discussion of these aspects see Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011).

To explore the accuracy of that approach for flow lines with limited buffer capacity (but without CONWIP control), Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011) analyzed a large and systematically created set of flow lines both via DES and the LP approach. That earlier paper also presents a detailed discussion of the errors induced by simulating a flow line that operates in continuous time within a discrete-time linear program, see Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011). The results showed that the method has a reasonable degree of accuracy unless buffers are very small and/or effective processing times (including possible repair times of unreliable machines) exhibit a high degree of variability with a coefficient of variation greater than 1.

2.2 Performance evaluation of CONWIP systems via linear programming

To model a CONWIP flow line system within a linear program, the following assumptions are made:

- A single product type is produced by the flow line. (This can represent a constant mixture of products for which the moments of the processing time can be calculated.)
- The production system contains a cyclic transportation system.
- There is a constant number, *pal*, of workpieces in the system due to a fixed number of pallets or PACs.
- The production capacity, c_{kt}, for each station k and period t is a realization of a stochastic count process.
- A production quantity, Q_{kt}, at a station during a period can either be further processed at the

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Table 1: Notation for the linear program

Indices	
$k = 1, .., K$	Workstations
$t = 1, .., T$	Periods
Input data	
b_k	Number of buffer spaces available behind station k
c_{kt}	Capacity, maximum number of workpieces that can be processed at station k in period t, provided that station k is neither blocked nor starved
pal	Fixed number of workpieces in the system (pallets or PACs)
T_0	Number of warm-up periods
Non-negative decision variables	
PR	Production rate estimate
Q_{kt}	Production quantity of station k in period t
$Y0_k$	Initial inventory behind station k
Y_{kt}	End-of-period inventory behind station k in period t

next work station during the next period or be stored in the downstream buffer.

- The buffer behind work station k can hold up to b_k workpieces.
- Transportation times as well as (un)loading times of pallets are negligible.

Note that a CONWIP flow line with limited buffer capacities behaves exactly like one with unlimited buffer capacities if the smallest buffer in the line is large enough to hold all workpieces circulating in the line.
A simple approach to evaluate the performance of such a system using linear programming is to maximize the production rate estimate for a given number of workpieces in the system. The constraints which have to be respected concern the inventory stored in the system, the production quantity (capacity given by the production system), the buffer space, and the number of workpieces used in the system.
Given the notation in Table 1 and the indicator function

$$(1)\quad \mathbb{1}_{\{x\}} = \begin{cases} 1, & \text{if } x \text{ is true} \\ 0, & \text{otherwise,} \end{cases}$$

the linear programming model can be stated as follows:

$$(2)\quad \text{Max } PR = \frac{1}{T - T_0} \cdot \sum_{t=T_0+1}^{T} Q_{Kt}$$

with respect to

$$(3)\quad Y0_k \cdot \mathbb{1}_{\{t=1\}} + Y_{k,t-1} \cdot \mathbb{1}_{\{1<t\leq T\}} + Q_{kt} = Y_{kt} + Q_{k+1,t+1} \cdot \mathbb{1}_{\{k<K,t<T\}} + Q_{1,t+1} \cdot \mathbb{1}_{\{k=K,t<T\}} \qquad \forall k,\ t$$

$$(4)\quad Q_{kt} \leq c_{kt} \qquad \forall k,\ t$$

$$(5)\quad Y_{kt} \leq b_k \qquad \forall k,\ t$$

$$(6)\quad Y0_k \leq b_k \qquad \forall k$$

$$(7)\quad \sum_{k\in K} \left(Y0_k + Q_{k,1}\right) = pal \qquad \forall t$$

$$(8)\quad \sum_{k\in K} \left(Y_{kt} + Q_{k,t+1}\right) = pal \qquad \forall t \setminus \{T\}$$

$$(9)\quad Y_{kt}, Y0_k, Q_{k,t} \geq 0 \qquad \forall k,\ t$$

$$(10)\quad PR \geq 0$$

The objective function (2) maximizes the production rate estimate at the last work station K. The production rate PR is determined by dividing the total production of the last work station K in periods $T_0 + 1$ to T by the length of that time span. Equations (3) are classical inventory balance equations. For each work station k and period t, the end-of-period inventory of the previous period $t-1$ plus the production quantity of the current period equals the current end-of-period inventory plus the production quantity of the following period $t+1$ at the next work station. Note that this "next" work station of the last work station K is station 1. Restrictions (4) state that the number of workpieces processed at station k must not exceed the maximum period-specific capacity c_{kt} from the count process described in Section 2.1. The inventory to be stored behind station k must not exceed the number of buffer spaces b_k as stated in Restrictions (5) and (6). Equations (7) and (8) ensure that the number of workpieces within the system meets the

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given total CONWIP level, *pal*, during each period. From a mathematical point of view, Equations (8) are redundant because they are already implied by Equations (3) in combination with Equation (7). Considering the case of $k = K$ and $t > 1$, Equations (3) turn to $Y_{K,t-1} + Q_{K,t} = Y_{K,t} + Q_{1,t+1}$. This is equivalent to $Y_{K,t-1} - Y_{K,t} + Q_{K,t} = Q_{1,t+1}$. The left-hand side represents the number of finished workpieces which leave the CONWIP line in period t. According to the CONWIP protocol, the same number of workpieces has to be sent to machine 1 in period $t + 1$. As we already guarantee the proper initial inventory with Equations (7), Equations (8) are redundant, but help to explain the logic of the model. Last, all decision variables must be non-negative, see Restrictions (9) and (10).

2.3 Optimization-oriented models

The basic (evaluation) model presented in Subsection 2.2 can be extended in different ways.

1. In the formulation given above, the number of workpieces *pal* in the system is assumed to be a parameter, determined by the number of pallets. To be able to optimize this number, the parameter *pal* has to be replaced by a non-negative decision variable *PAL* in Equations (7) and (8). The resulting new constraints are given in Restrictions (11) to (13).

$$(11)\quad \sum_{k\in K} \left(Y0_k + Q_{k,1}\right) = PAL \qquad \forall t$$

$$(12)\quad \sum_{k\in K} \left(Y_{kt} + Q_{k,t+1}\right) = PAL \qquad \forall t$$

$$(13)\quad PAL \geq 0$$

   Note that via this modeling approach, a single discrete-time simulation run within a linear program based on a sample of realized production capacities c_{kt} can be used to optimize a stochastic CONWIP flow line, here with respect to the production rate.

2. The model presented so far yields a production rate estimate for the system characterized by a given CONWIP level *pal* or a production rate maximizing CONWIP level *PAL*. In a more economic perspective, it is interesting to find the profit-maximizing number of workpieces in the system. The solution of such a model depends on the cost of capital tied up in the work-in-process and the value of the produced workpieces. Let *hc* denote the holding cost per product unit and time period and *gm* the gross margin per product unit. Now the objective is to maximize the profit per time unit which depends on the gross margin of the finished workpieces and the holding cost of all workpieces in the system:

$$(14)\quad \text{Max } Profit = gm \cdot \left(\frac{1}{T - T_0} \cdot \sum_{t=T_0+1}^{T} Q_{Kt}\right) - hc \cdot PAL$$

   The Restrictions (3) to (6) and (9) to (13) remain the same.

3. Similar to the first extension, the model can be used to optimize the distribution of buffers for a given total buffer capacity b_{tot}. The parameter b_k for local buffer capacities in Restrictions (5) and (6) has to be replaced by non-negative decision variables X_k as shown in Restrictions (15), (16) and (18):

$$(15)\quad Y_{kt} \leq X_k \qquad \forall k, t$$

$$(16)\quad Y0_k \leq X_k \qquad \forall k$$

$$(17)\quad \sum_{k\in K} X_k = b_{tot}$$

$$(18)\quad X_k \geq 0$$

   Furthermore, Constraint (17) has to be added to guarantee that the total number of buffers allocated behind the different stations k in the system meets the total number of buffers available b_{tot}

Other modifications of this generic model are possible as well.

3 Numerical results

3.1 Accuracy of production rate estimates

3.1.1 Outline of the numerical study

In order to evaluate the accuracy and the numerical effort of our method, we performed a numerical study considering CONWIP lines. The design of

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this study is based on the results presented by Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011). One result of that paper is that an average processing rate of about 1.0 workpieces per (discrete) time unit combined with a length of a "simulation run" of 10,000 discrete-time units yields a reasonable balance between the sampling frequency, the number of sampled events and the size of the matrix of the linear program embedding such a "simulation run". A further result of that paper is that the method is not accurate for very small buffer sizes and/or coefficients of variation of the effective processing times greater than 1.

As the measure of accuracy, we use the relative deviation of the production rate estimates of the discrete-time linear program from the "true value" (gained by an extremely long and very precise discrete-event simulation). We compare the results of our method to those obtained from a discrete-event simulation model coded in C, see Helber (1999: 114). The LP models from Sections 2.2 and 2.3 were implemented in GAMS. Cplex 11.0.0 was used on a Dual Core Pentium IV machine with 2.8 GHz and 2 GB RAM to solve the models.

We investigate the impact of the following aspects of the problem instances on the accuracy of our method:

- Number of stations
- Number of buffer spaces for each buffer between the machines in the flow line
- Average processing rates at the machines
- Location of the bottleneck (if any) in the line
- Variability of the effective processing times
- Number of pallets (relative to the number of spaces for pallets in the system)
- Exogenously given even distribution of buffer spaces vs. endogenously determined (production rate maximizing) allocation of buffer spaces.

For reasons of transparency we first discuss these aspects briefly: We expect to find larger deviations for increasing **numbers of stations**, given the results for open lines in Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011). As buffers reduce blocking and starving and our method only approximates the true movement of finished workpieces in Equations (3), we expect to achieve more precise results for problem instances with larger **numbers of buffers**.

The number of periods in the discrete-time linear program and the **processing rates** determine the total production quantity within the solution of the linear program. To create comparable conditions in our experiments, we set the number of simulated periods (after a warm-up period) and the processing rate of the machines in such a way that expected numbers of approximately 10,000 workpieces could be processed by the line after an initial warm-up period of $T_0 = 500$ periods. Let μ^* denote the rate at which the bottleneck machine of the line operates. Then the number of periods in the discrete-time linear program was determined as follows:

$$(19)\quad T = T_0 + T_1 = 500 + \lceil 10,000 \frac{1}{\mu^*} \rceil$$

Given the circular structure of CONWIP flow lines, we expect that the accuracy of the method does not depend on the location of a bottleneck in such a line.

With respect to the **variability of processing times**, we conjecture to find an increasing accuracy with decreasing variability as we did in the study for open flow lines, see Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011).

As the number of workpieces in a CONWIP line is restricted by the number of pallets, we investigate their influence controlled by a **pallets factor**. It calculates the number of pallets based on the number of buffer spaces plus the number of spaces at the work stations. For a given pallets factor, pf, the number of pallets in the system is therefore computed as follows:

$$(20)\quad pal = pf \cdot \left(K + \sum_{k=1}^{K} b_k \right)$$

The previous experiments presented in Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011) revealed that the accuracy of the production rate estimates for open flow lines is similar for an exogenously given (even) buffer allocation and an endogenously determined (uneven) **buffer allocation**. We wanted to check if this holds for CONWIP lines as well.

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Table 2: Test Bed for the analysis of CONWIP lines (2430 cases)

Parameter type	No. cases	Parameter value per case
Number of stations	3	5, 7, 9
Buffer spaces per buffer	3	4, 8, 16
Base processing rates	3	0.5, 1.0, 2.0
Bottleneck factor	3	(f. m.: 0.9; o. m.: 1.0), (balanced line, all machines 1.0), (l. m.: 0.9; o. m.: 1.0)
Processing time variability (SCV)	3	0.25, 0.5, 1.0
Pallets factor	5	0.2, 0.35, 0.5, 0.65, 0.8
Buffer allocation	2	even vs. production-rate maximizing

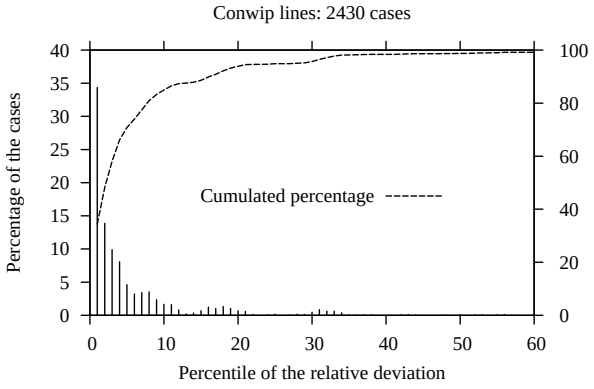
"f. m." means "first machine", "l. m." means "last machine","o. m." means "other machines"

For all the parameter types described, we systematically explored a range of parameter values, to find out under which conditions the method yields reasonably precise production rate estimates.

3.1.2 Comparison with continuous time simulation results

To evaluate the performance of our method for CONWIP lines, we used a test bed consisting of all possible combinations of the parameters described in Table 2. We compared the results of our method to those computed with a discrete-event simulation for the test bed consisting of 2430 (= 3·3·3·3·3·5·2) cases. Given the results of the method for open flow lines, we considered in this paper minimum buffer sizes of 4 and maximum squared coefficients of variation (SCV) of processing times of 1.0. (The SCV of a random variable is the squared ratio of its standard deviation to its expected value.) The last line in Table 2 indicates that for each line we first evaluated a given even distribution of the buffer spaces in the line and then sought the production-rate maximizing buffer allocation as described in Sections 2.2 and 2.3.
Figure 3 shows a diagram with the frequencies of absolute values of relative deviations of production rate estimates obtained by the LP approach from

Figure 3: Percentage of cases over relative deviations for all 2430 cases of CONWIP lines



those of the DES. The maximum relative deviation is about 60%, the mean value of the relative deviation is about 5.7%. It also reveals that in 70.78% of the cases there is a deviation of less than 5%. Considering the average of the relative deviation of the production rate estimate as shown in Tables 3 to 8, our method tends to underestimate the production rate. This is a consequence of our discrete-time approach which assumes in Equations (3) that workpieces always have to wait for the end of a period to move to the next work station.
To lay open the impact of the parameters listed in Table 2, their effect on the results is shown in the following Tables 3 to 8. We always report

- $RelDev = \frac{PR^{LP} - PR^{Sim}}{PR^{Sim}}$, the average of the relative deviation of the production rate estimate,
- *AbsRelDev*, the associated average over absolute values of relative deviations, and
- *CPU*, the time (in seconds) to solve the linear program.

The upper part of the tables is dedicated to the cases in which the buffer allocation is exogenously given and buffer spaces are evenly distributed. The lower part includes the results for the optimized (production rate maximizing) buffer allocation.

Against our expectations we did not observe a strong impact of the number of stations in the CONWIP line on the accuracy of the production rate estimate for our instances. Here the results

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Table 3: Impact of the number of stations in the line

Stations	5	7	9
Even buffer allocation			
RelDev [%]	-5.0	-5.0	-5.3
AbsRelDev [%]	5.4	5.7	5.9
CPU [sec.]	18.7	32.4	49.8
Optimized buffer allocation			
RelDev [%]	-4.7	-4.8	-5.3
AbsRelDev [%]	5.7	5.9	5.8
CPU [sec.]	30.7	52.2	80.9

Table 4: Impact of the number of buffer spaces per buffer

Buffer spaces per buffer	4	8	16
Even buffer allocation			
RelDev [%]	-10.9	-3.5	-1.0
AbsRelDev [%]	11.7	4.1	1.3
CPU [sec.]	31.4	33.6	35.9
Optimized buffer allocation			
RelDev [%]	-10.7	-3.5	-0.6
AbsRelDev [%]	11.8	4.0	1.5
CPU [sec.]	53.5	53.7	56.6

differ from those for open flow lines in Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011) where the accuracy decreased with an increasing number of stations. The results in Table 3 indicate that the CPU time rises as the size of the LP grows with the number of stations.

The results in Table 4 reveal that the accuracy of the method increases with an increasing number of buffer spaces. The range of chosen parameter values does not seem to have a strong influence on the CPU times.

Table 5 shows the impact of the base processing rate. For the production of one workpiece per time unit, the method appears to yield the best results. We cannot yet explain this observation. The CPU time decreases with increasing base processing rates because the number of periods ($T - T_0$) after the T_0 warm-up periods decreases (see Section 3.1.1).

Table 5: Impact of the base processing rate

Base processing rate	0.5	1.0	2.0
Even buffer allocation			
RelDev [%]	-6.1	-3.7	-5.5
AbsRelDev [%]	6.5	4.0	6.6
CPU [sec.]	53.0	32.1	15.8
Optimized buffer allocation			
RelDev [%]	-6.0	-3.3	-5.5
AbsRelDev [%]	6.5	4.3	6.6
CPU [sec.]	92.3	48.4	23.1

Table 6: Impact of the bottleneck location

Bottleneck location	f.m.	b. l.	l.m.
Even buffer allocation			
RelDev [%]	-5.0	-5.3	-5.0
AbsRelDev [%]	5.7	5.8	5.7
CPU [sec.]	35.3	30.4	35.2
Optimized buffer allocation			
RelDev [%]	-4.8	-4.9	-5.1
AbsRelDev [%]	5.8	6.1	5.6
CPU [sec.]	56.8	48.0	59.0

"f. m." means "first machine", "b. l." means "balanced line", "l. m." means "last machine"

Table 7: Impact of the squared coefficient of variation

SCV	0.25	0.5	1.0
Even buffer allocation			
RelDev [%]	-2.9	-4.6	-7.8
AbsRelDev [%]	3.8	5.1	8.3
CPU [sec.]	37.6	33.8	29.5
Optimized buffer allocation			
RelDev [%]	-2.8	-4.1	-7.9
AbsRelDev [%]	3.7	5.5	8.1
CPU [sec.]	58.2	54.2	51.4

As expected, the location of a bottleneck in a CONWIP line does not seem to have a strong influence on the accuracy of the results, see Table 6.

Table 7 reveals the strong impact of the variability of the effective processing times. The lower the SCV, the more accurate the production rate estimates are. The CPU times decrease as the SCV increases. We conjecture that a higher volatility of the sampled production capacities c_{kt} leads to more "extreme" restrictions of the solution space so that the optimum of the LP can be found more quickly.

Table 8 indicates that the relative number of pallets in the system (related to the number of buffers and stations) is very important. In cases with a low (0.2) or a high pallets factor (0.8), the results are less accurate than in the other cases.

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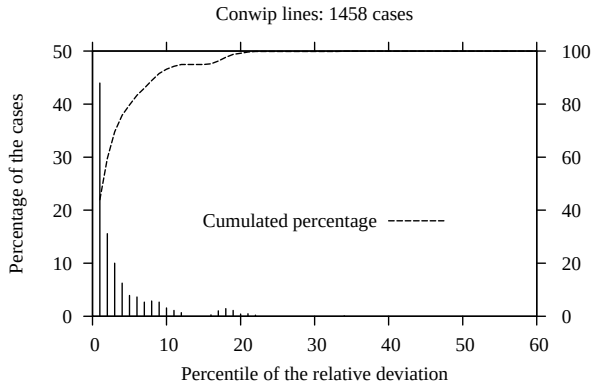
Table 8: Impact of the pallets factor

Pallets factor *pf*	0.2	0.35	0.5	0.65	0.8
Even buffer allocation					
RelDev [%]	-7.7	-3.8	-2.3	-2.6	-9.2
AbsRelDev [%]	8.5	4.0	2.4	3.0	10.5
CPU [sec.]	33.7	34.9	34.9	34.4	30.2
Optimized buffer allocation					
RelDev [%]	-7.9	-3.8	-2.2	-2.3	-8.4
AbsRelDev [%]	8.4	3.9	2.4	3.3	10.9
CPU [sec.]	49.1	53.8	62.3	56.0	51.8

Figure 4: Percentage of cases over relative deviations for the subset of 1458 cases



For a low pallets factor, the system has relatively few pallets so that starving occurs frequently. By the same token, a high pallets factor leads to frequent blocking. If the system is starved or blocked often, the discrete-time LP modeling "defect" that pallets can only move to the next station at the end of the period (see Equation (3)) becomes more important than if these events occur less often for medium pallets factors.

As this seems to be a major finding of the study we used the method again for a subset of the test bed shown in Table 2. We created this subset by eliminating the two parameter values 0.2 and 0.8 of the pallets factor. The results obtained for the remaining 1458 cases (= 3 · 3 · 3 · 3 · 3 · 3 · 2) are shown in Figure 4. The method yields much more accurate results for this subset of the test bed. Now the maximum relative deviation is about 20%, the mean value of the relative deviation is 3.18%. In 79.7% of the cases, there is a deviation of less than 5%.

The results show that our method works well for a wide range of relevant parameter settings.

3.2 Optimizing CONWIP levels via the linear programming approach

In the previous section, we asked for the accuracy of the production rate estimates for given numbers of pallets. Now we treat the number of pallets as a decision variable. To study the problem of optimizing inventory levels with respect to the production rate or the profit, we consider a balanced five-machine CONWIP line as depicted in Figure 1. The gross margin *gm* per workpiece is 100 monetary units.

With respect to the inventory cost parameter we study two cases. In the first case, we assume that the holding cost *hc* per unit and (discrete) time period is 0.1 monetary units. Note that this first inventory cost parameter already implies that holding inventory in the flow line is very costly. In addition, we study a second case with an extremely high inventory cost parameter of 1.0 monetary units per material unit and time period. (The case of $hc = 0$ only leads to a re-scaling of the graphs for the production rate in Figures 5 to 7.) We chose the extreme value of $hc = 1$ to show cases with a distinct peak in the function of the profit over the number of pallets even in cases with unlimited local buffer sizes. The average processing rates of the machines are 1.0 workpieces per time unit and the squared coefficients of variation of the processing times at all machines in the line are either 0.1, 0.5, or 1.0, respectively, as for these values the method worked well for open flow lines, see Helber, Schimmelpfeng, Stolletz, and Lagershausen (2011). With respect to buffer sizes, we consider two cases: In the first case, we assume a buffer capacity of 10 workpieces behind each machine. In the second case, we set all the buffer capacities to 100. The CONWIP level varies from 1 to 50.

If each buffer can hold up to 10 workpieces, blocking can occur for CONWIP levels above 10 workpieces. However, deadlock cannot occur for the assumed maximum CONWIP level of 50 workpieces as machines and buffers can (together) hold up to 55 workpieces. The other case with buffer capacities of 100 workpieces behind each machine and a maximum CONWIP level of 50 workpieces models the infinite buffer capacity case as blocking cannot occur.

For each buffer capacity case and CONWIP level, we determine a production rate estimate via our LP method and via the discrete-event simulation

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(DES) in continuous time. Based on these production rate estimates for different CONWIP levels, the short-term profit is computed as specified in Equation (14).
We first vary the CONWIP level from 1 to 50 to show the production rate and the profit as a function of the number of pallets. Then we ask how reliably our LP approach can find the number of pallets that maximizes the production rate or the profit. For that purpose, we repeatedly solved the models for 10 different realizations of the simulated processing times.
The graphs for the production rate estimates as determined via our method and via a discrete-event simulation are depicted in Figures 5 to 7. They show that the production rate decreases as processing time variability increases and that CONWIP lines with unlimited buffer capacities are more productive than those with limited buffer capacities. They also show that for peak production rates, the results of the discrete-time model (LP) are very close to those of the continuous time simulation (Sim).
The results for the profit in Figures 8 to 13 show a very similar picture. The CONWIP line reaches its peak profitability in situations where blocking and starving rarely occur. Under these conditions, however, our method is apparently relatively accurate.
Note that the profit functions in Figures 8 to 13 exhibit a—from a practical point of view—extremely nice feature: As the variability of the effective processing time increases, the profit function becomes flatter around its maximum. While our method yields less accurate production rate estimates as the variability increases, the profit estimates are therefore still relatively exact and the solutions can be expected to be close to optimal. It should also be noted that our method always slightly underestimates the profit associated with a given CONWIP level as it tends to underestimate the production rate, see above.
In the last part of the numerical experiment, we address the question of how reliably our method can find the number of pallets that maximizes the production rate or the profit for a given line without enumerating all pallet levels as shown in Figures 5 to 13. We also want to quantify how precise the production rate or profit estimate is around the "true optimum". For that purpose we studied the same cases that led to the results presented in Figures 5 to 13 and now make the number of pallets *PAL* a decision variable.
Remember that each single optimization for the linear program is based on different realizations of random variables for production capacities. Therefore, each optimization run leads to different estimates of the production rate and/or profit associated with a particular line. It also leads to different estimates of the respective optimal number of pallets. For this reason, we performed 10 independent optimization runs for each of the six systems and both objectives (production rate or profit maximization), leading to specific estimates of the optimal number of pallets. We then asked

- how strongly the estimated optimum objective function value (from the discrete-time LP) deviates from the "true" optimum as obtained via the DES and
- how close the "optimal" number of pallets *PAL* as determined via the LP comes to the "true" production rate or profit-maximizing number of pallets and
- how much of the true optimum of the respective objective function is sacrificed if the pallet numbers as determined via the LP are implemented.

The results are presented in Tables 9 to 11. Note that maximizing the production rate as a function of the number of pallets is only a reasonable objective for the case of limited buffer capacities. Therefore we do not report results for unlimited buffer capacity.
For each squared coefficient of variation we first report the maximum of the considered objective value from the continuous time simulation, i.e., in Table 9 the production rate maximum PR^{Sim} from the continuous time simulation and the corresponding "true" optimal number of pallets PAL^{Sim}.
The lower part of the table reports results from our linear programming approach. We start with the range of production rate estimates PR^{LP} over the 10 independent replications of the LP optimization. The respective range of relative deviations from the true optimum is labeled *RelDev*1. We next present the range of pallet numbers PAL^{LP} that were considered to be "optimal" within the 10 replications of the optimization. In the bottom part of the table we finally report for that range of pallet numbers the corresponding range of production

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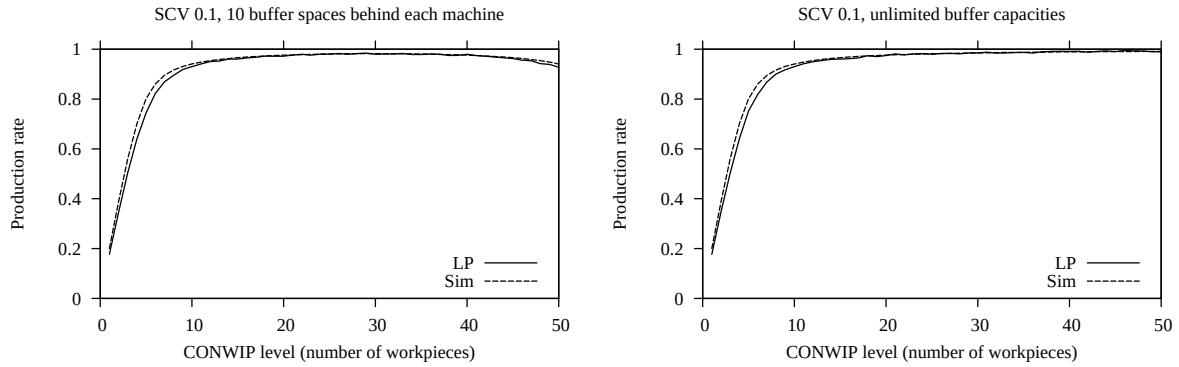
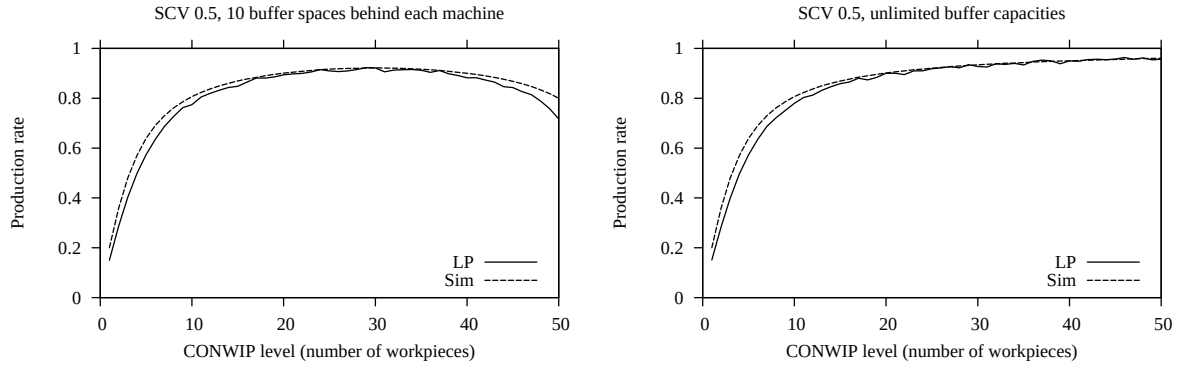
Figure 5: Production rate for small resp. infinite buffers and low variability



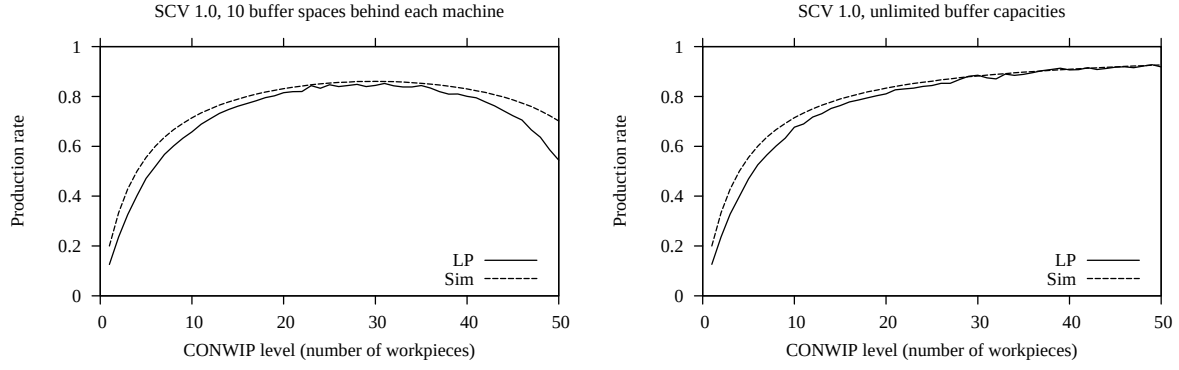
(a) Small buffers (b) Infinite buffers

Figure 6: Production rate for small resp. infinite buffers and moderate variability



(a) Small buffers (b) Infinite buffers

Figure 7: Production rate for small resp. infinite buffers and high variability



(a) Small buffers (b) Infinite buffers

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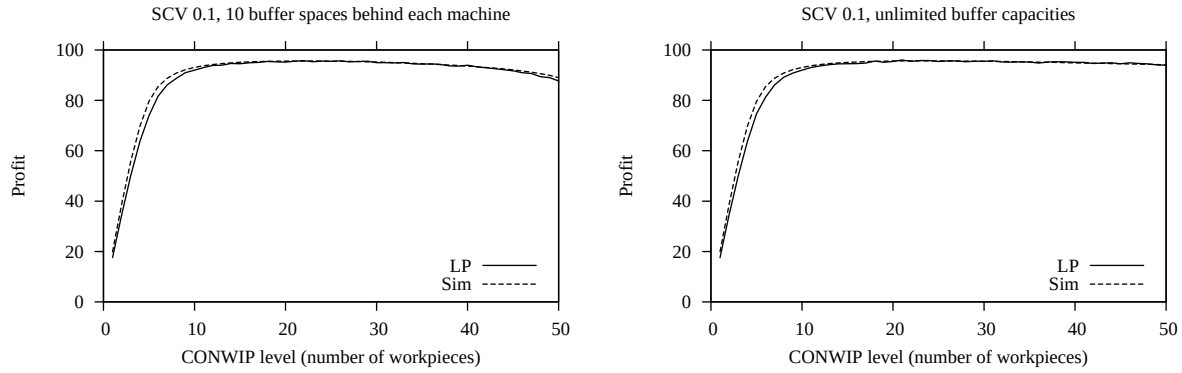
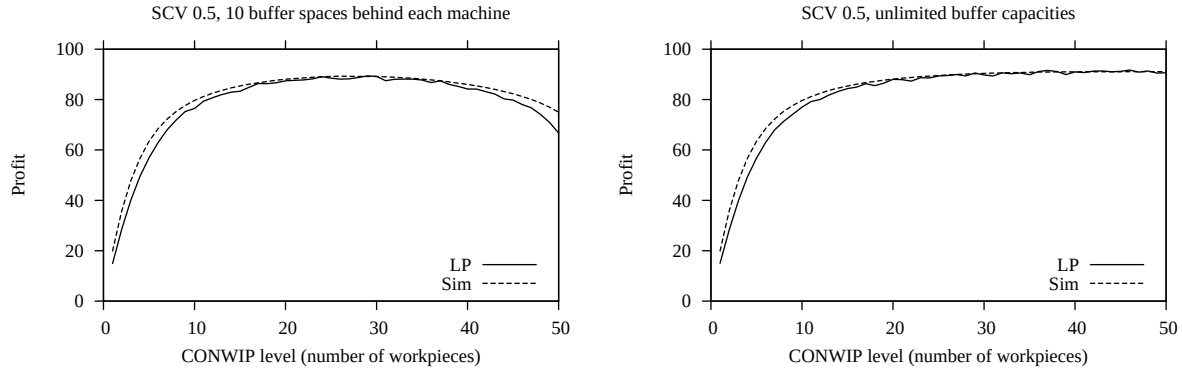
Figure 8: Profit for small resp. infinite buffers and low variability (*hc* = 0.1)



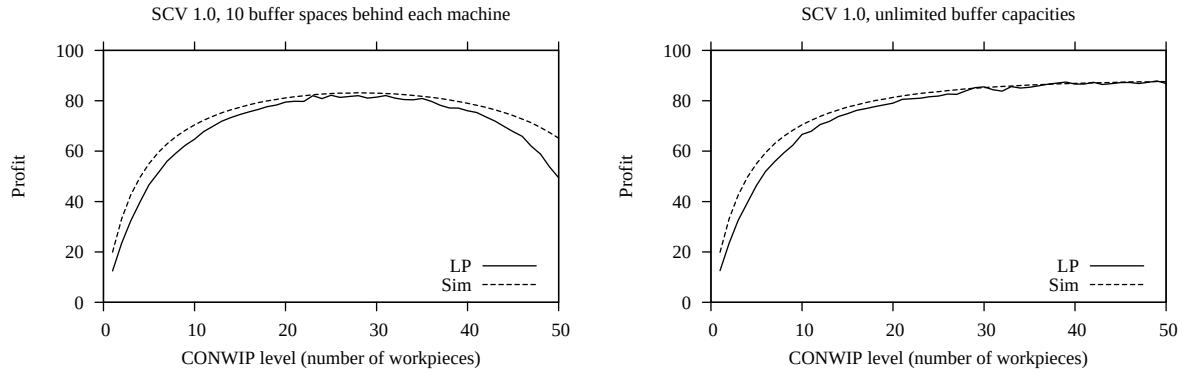
(a) Small buffers (b) Infinite buffers

Figure 9: Profit for small resp. infinite buffers and moderate variability (*hc* = 0.1)



(a) Small buffers (b) Infinite buffers

Figure 10: Profit for small resp. infinite buffers and high variability (*hc* = 0.1)



(a) Small buffers (b) Infinite buffers

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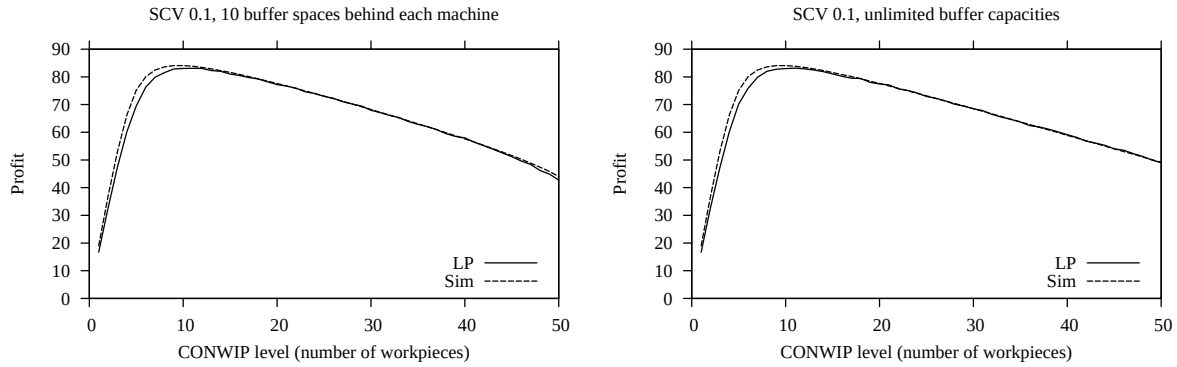
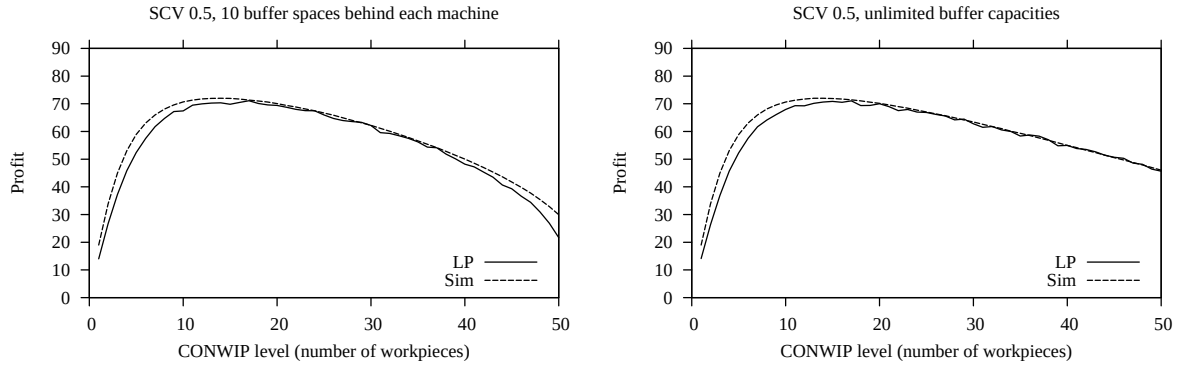
Figure 11: Profit for small resp. infinite buffers and low variability (*hc* = 1.0)



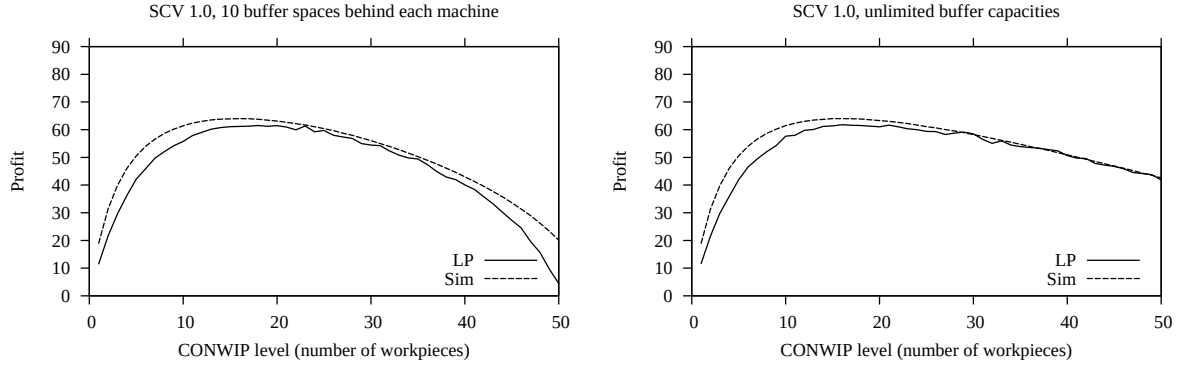
(a) Small buffers (b) Infinite buffers

Figure 12: Profit for small resp. infinite buffers and moderate variability (*hc* = 1.0)



(a) Small buffers (b) Infinite buffers

Figure 13: Profit for small resp. infinite buffers and high variability (*hc* = 1.0)



(a) Small buffers (b) Infinite buffers

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Table 9: Production rate maximization for finite buffer cases: Accuracy of the estimated optimal number of pallets

SCV	0.1	0.5	1.0
Continuous time simulation:			
$PR^{Sim}(PAL^{Sim})$	0.982	0.922	0.861
PAL^{Sim}	30	30	30
LP-Approach:			
$PR^{LP}(PAL^{LP})$	0.980 ··· 0.983	0.912 ··· 0.925	0.841 ··· 0.854
*RelDev*1	-0.2% ··· 0.1%	-1.1% ··· 0.3%	-2.3% ··· -0.8%
PAL^{LP}	29 ··· 31	28··· 30	28··· 30
$PR^{Sim}(PAL^{LP})$	0.982 ··· 0.982	0.920 ··· 0.922	0.860 ··· 0.861
*RelDev*2	≈ 0%	-0.2% ··· 0%	-0.1% ··· 0%

Table 10: Profit maximization for finite buffer cases: Accuracy of the estimated optimal number of pallets

SCV	0.1	0.5	1.0
Continuous time simulation:			
$Profit^{Sim}(PAL^{Sim})$	84.1	72.0	64.0
PAL^{Sim}	10	14	16
LP-Approach:			
$Profit^{LP}(PAL^{LP})$	82.7 ··· 83.4	70.3 ··· 71.5	61.5 ··· 62.4
*RelDev*1	-1.7% ··· -0.8%	-2.4% ··· -0.7%	-0.2% ··· -2.5%
PAL^{LP}	10 ··· 11	15 ··· 16	17 ··· 18
$Profit^{Sim}(PAL^{LP})$	84.1 ··· 83.8	71.9 ··· 71.7	63.9 ··· 63.7
*RelDev*2	0% ··· -0.4%	-0.1% ··· -0.4%	-0.2% ··· -0.5%

Table 11: Profit maximization for infinite buffer cases: Accuracy of the estimated optimal number of pallets

SCV	0.1	0.5	1.0
Continuous time simulation:			
$Profit^{Sim}(PAL^{Sim})$	84.1	72.0	64.0
PAL^{Sim}	9	13	15
LP-Approach:			
$Profit^{LP}(PAL^{LP})$	82.9 ··· 83.3	69.9 ··· 71.1	60.8 ··· 63.2
*RelDev*1	-1.4% ··· -1.0%	-2.9% ··· -1.3%	-5.0% ··· -1.3%
PAL^{LP}	10 ··· 11	all 15	17 ··· 18
$Profit^{Sim}(PAL^{LP})$	84.1 ··· 83.8	all 71.9	63.9 ··· 63.8
*RelDev*2	0% ··· -0.4%	all -0.1%	-0.2% ··· -0.3%

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rates $PR^{Sim}(PAL^{LP})$ from the DES and the respective range of relative deviations from the true optimum $PR^{Sim}(PAL^{Sim})$. This last number *RelDev*2 indicates by how many percent we actually miss the optimum value by setting the inventory level *PAL* via our LP method. The structure of Tables 10 and 11 related to profit maximization is identical. Tables 9 to 11 show that in our examples we never miss the optimum objective function value by more than about 0.5%. This is due to the fact that both the production rate and the profit are flat around the optimum number of pallets, see Hopp and Spearman (2000: 358).

Some results from these tables deserve a more detailed discussion. Note that in Table 9, the number of pallets that maximizes the production rate is always 30, for any variability of the effective processing times. This is plausible as in the balanced five-machine line with identical buffer sizes, five workpieces are required at the machines and the remaining 25 workpieces use exactly 50% of the $5 \cdot 10 = 50$ buffer spaces in the line and make blocking and starving of machines equally likely, and hence maximize the production rate.

As we expected, maximum production rates and profit levels decrease as the variability of processing times increases. It is also interesting to note that both the maximum profit and the corresponding number of pallets in Tables 10 and 11 are almost identical. This has a potentially important managerial implication. If pallets are expensive and one seeks a profit-maximizing configuration of a flow line, it might be worthwhile to study the infinite buffer case first and determine an estimate of the optimal number of pallets. This helps to set an upper bound on the number of buffer spaces that may be required between any two adjacent machines and thus speed up the search process for a good buffer allocation.

3.3 Optimizing the buffer allocation

For open (i.e., non-CONWIP) flow lines that face unlimited raw material at the first station and unlimited demand at the the last station, some structural insights about the production-rate maximizing allocation of buffers have been established, see Hillier, So, and Boling (1993): If all stations are stochastically identical, the production-rate maximizing allocation of buffers resembles the shape of a bowl turned upside down. Stations in the interior of the line have the highest risk of being starved or blocked and hence attract more buffer spaces than the first (or the last) stations. If a particular station is a clear bottleneck, this station attracts many buffer spaces to protect the valuable bottleneck resource from blocking or starving. However, many different allocations of buffers can lead to a very similar performance of the line and therefore very precise simulations or analytical performance evaluation methods are required to find the optimal allocation. If, as in our numerical study, only the production of 10,000 workpieces is simulated, this is a rather rough simulation and hence one can only expect to find a reasonable, but not the truly optimal allocation of buffers. In addition, we terminated the solution of the linear program when the integrality gap was below 0.5%, which also induces some inaccuracy. However, from a practical point of view, the value of finding the true optimum may be limited, as the production rate differences of similar buffer allocations are often relatively small.

To show this effect, consider out of the test bed in Table 2 of Section 3.1.2 the case of the five-machine line with 8 buffer spaces per line, a base processing rate of 1.0 workpieces per period and a bottleneck with only 90% capacity at the last station of the line. Remember that we have already shown that our method is most accurate for medium CONWIP levels that also lead to the highest production rate, see Table 8 and the results in Section 3.2. In Tables 12 to 14 we show for this five-machine line the results for the even and the optimized buffer allocation for medium pallet factors of 0.35, 0.5 and 0.65. In a five-station line with 8 buffer spaces behind each station, the system can hold 45 workpieces. The three different pallet factors of 0.35, 0.5 and 0.65 lead to pallet numbers between $\lceil 0.35 \cdot (40 + 5) \rceil = 15$ and $\lceil 0.65 \cdot (40 + 5) \rceil = 29$. For squared coefficients of variation of 0.25, 0.5 and 1.0, we hence consider pallet numbers of 15, 22 and 29 with either an even or (immediately below) an optimized buffer allocation. We report the production rate estimate from the linear program (PR^{LP}), the average as well as (in brackets) the 95% confidence interval of the production rate estimate PR^{Sim} from the discrete-event simulation and the relative deviation of the estimates.

The tables show that if the buffer allocation is optimized, the number of buffer spaces in front and behind the fifth station (i.e., the bottleneck)

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Table 12: Even vs. optimized buffer allocation (SCV=0.25)

Pallets	Buffer allocation	PR^{LP}	PR^{Sim}	*RelDev*[%]
15	(8, 8, 8, 8, 8)	0.878330	0.885587 [0.885157, 0.886017]	-0.82
15	(8, 6, 7, 8, 11)	0.883459	0.885678 [0.884945, 0.886411]	-0.25
22	(8, 8, 8, 8, 8)	0.891109	0.894996 [0.894268, 0.895724]	-0.43
22	(6, 6, 6, 9, 13)	0.893089	0.896015 [0.895339, 0.896691]	-0.33
29	(8, 8, 8, 8, 8)	0.891379	0.894565 [0.893972, 0.895157]	-0.36
29	(6, 8, 6, 10, 10)	0.898308	0.895494 [0.894773, 0.896216]	0.31

Table 13: Even vs. optimized buffer allocation (SCV=0.5)

Pallets	Buffer allocation	PR^{LP}	PR^{Sim}	*RelDev*[%]
15	(8, 8, 8, 8, 8)	0.823704	0.841433 [0.840882, 0.841983]	-2.11
15	(8, 7, 7, 10, 8)	0.834773	0.841598 [0.841012, 0.842184]	-0.81
22	(8, 8, 8, 8, 8)	0.866811	0.86786 [0.867266, 0.868454]	-0.12
22	(6, 6, 7, 10, 11)	0.863031	0.869983 [0.869487, 0.870479]	-0.80
29	(8, 8, 8, 8, 8)	0.850612	0.866426 [0.865676, 0.867175]	-1.83
29	(7, 8, 7, 9, 9)	0.854212	0.868125 [0.867548, 0.868703]	-1.60

Table 14: Even vs. optimized buffer allocation (SCV=1.0)

Pallets	Buffer allocation	PR^{LP}	PR^{Sim}	*RelDev*[%]
15	(8, 8, 8, 8, 8)	0.736861	0.767438 [0.766885, 0.76799]	-3.98
15	(7, 8, 7, 9, 9)	0.742351	0.768198 [0.767312, 0.769083]	-3.36
22	(8, 8, 8, 8, 8)	0.796076	0.807794 [0.8069, 0.808689]	-1.45
22	(7, 8, 6, 10, 9)	0.788427	0.807912 [0.807013, 0.808812]	-2.41
29	(8, 8, 8, 8, 8)	0.779698	0.804565 [0.803814, 0.805315]	-3.09
29	(8, 7, 8, 9, 8)	0.781677	0.805266 [0.804573, 0.805959]	-2.93

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is slightly larger than in the rest of the system, i.e., the bottleneck station attracts more buffer spaces than the non-bottleneck stations. However, the impact of the buffer allocation optimization on the production rate is only minimal. In most cases it is not even statistically significant as most of the 95% confidence intervals of the production rate estimates from the very precise discrete-event simulation overlap. Our impression is that the total number of buffer spaces and the number of pallets in the system is very important for the production rate of the system, but the value of fine-tuning the buffer allocation appears to be limited.

4 Conclusion and further research

In this paper we analyzed the performance evaluation and optimization of stochastic flow lines under the CONWIP protocol. We used a simple linear program that models an entire simulation run of the closed-loop system in discrete time. This way, it is possible to evaluate and optimize the production rate and/or short-term profit of the CONWIP system.

Our approach offers the optimization power of (mixed-integer) linear programming in combination with the flexibility of stochastic simulation with respect to probability distributions of stochastic processing times. It avoids the disadvantages of the established approaches (e.g., difficulty to optimize based on DES, special knowledge requirements and restrictive assumptions within queueing models). The accuracy of the method depends especially on the variability of the processing times and the number of pallets in the CONWIP line.

In particular for profit-maximizing CONWIP levels, the approach appears to be remarkably accurate unless buffers are very small and/or effective processing times are highly variably. As hardware and software continue to become more and more powerful, it will be possible to study longer lines and systems with higher degrees of variability of the effective processing times using our method. Our future work will address flow line configuration and design problems from an investment perspective.

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