

9. TEKNIK PENGINTEGRALAN

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9.1 Integral Parsial

Formula Integral Parsial :

$$\int u \, dv = uv - \int v \, du$$

Cara : pilih u yang turunannya lebih sederhana

Contoh : Hitung $\int x e^x \, dx$

misal $u = x$, maka $du = dx$

$$dv = e^x \, dx \Rightarrow v = \int e^x \, dx = e^x$$

sehingga

$$\int x e^x \, dx = x e^x - \int e^x \, dx = x e^x - e^x + C$$

Integral parsial dapat dilakukan lebih dari satu kali

Contoh Hitung $\int x^2 \sin x dx = -x^2 \cos x + \int x \cos x dx$

Jawab

Integral parsial

(i) Misal $u = x^2 \rightarrow du = 2x dx$

$dv = \sin x dx \rightarrow v = -\cos x$

(ii) Misal $u = x \rightarrow du = dx$

$dv = \cos x dx \rightarrow v = \sin x$

$= -x^2 \cos x + (x \sin x - \int \sin x dx)$

$= -x^2 \cos x + x \sin x + \cos x + C$

Ada kemungkinan integran (f(x)) muncul lagi diruas kanan

Contoh Hitung $\int e^x \cos x dx$

Integral parsial

Jawab : $\int e^x \cos x dx = e^x \sin x - \int e^x \sin x dx$

(i) Misal $u = e^x \rightarrow du = e^x dx$

$dv = \cos x dx \rightarrow v = \sin x$

(ii) Misal $u = e^x \rightarrow du = e^x dx$

$dv = \sin x dx \rightarrow v = -\cos x$

$= e^x \sin x - (-e^x \cos x + \int e^x \cos x dx) + C$

$= e^x \sin x + e^x \cos x - \int e^x \cos x dx + C$

Integral yang dicari
,bawa keruas kanan

$2 \int e^x \cos x dx = e^x \sin x + e^x \cos x + C$

$\int e^x \cos x dx = \frac{1}{2} (e^x \sin x + e^x \cos x) + C$

Soal latihan

Hitung

1. $\int_1^e \ln x \, dx$

2. $\int_1^e x \ln x \, dx$

3. $\int \ln(1 + x^2) \, dx$

4. $\int \sin^{-1} x \, dx$

5. $\int \tan^{-1} x \, dx$

6. $\int x \tan^{-1} x \, dx$

9.2 Integral Fungsi Trigonometri

Bentuk : $\int \cos^n x \, dx$ & $\int \sin^n x \, dx$

* Untuk n ganjil, Tuliskan :

$$\sin^n x = \sin x \sin^{n-1} x \quad \text{dan} \quad \cos^n x = \cos x \cos^{n-1} x$$

dan gunakan identitas $\sin^2 x + \cos^2 x = 1$

* Untuk n genap, Tuliskan :

$$\sin^n x = \sin^2 x \sin^{n-2} x \quad \text{dan} \quad \cos^n x = \cos^2 x \cos^{n-2} x$$

dan gunakan identitas $\cos 2x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$

Contoh Hitung

1. $\int \sin^3 x dx$

2. $\int \sin^4 x dx$

Jawab

1. $\int \sin^3 x dx = \int \sin^2 x \sin x dx = \int (1 - \cos^2 x) d(\cos x) = \cos x - \frac{1}{3} \cos^3 x + C$

2. $\int \sin^4 x dx = \int \sin^2 x \sin^2 x dx = \int \left(\frac{1 - \cos 2x}{2}\right) \left(\frac{1 - \cos 2x}{2}\right) dx$

$$= \frac{1}{4} \int (1 - 2 \cos 2x + \cos^2 2x) dx = \frac{1}{4} \left(\int dx - 2 \int \cos 2x dx + \int \frac{1 + \cos 4x}{2} dx \right)$$
$$= \frac{1}{4} x - \frac{1}{4} \sin 2x + \frac{1}{8} x + \frac{1}{32} \sin 4x + C = \frac{3}{8} x - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$$

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■ Bentuk $\int \sin^m x \cos^n x dx$

a). Untuk n atau m ganjil, keluarkan $\sin x$ atau $\cos x$ dan gunakan identitas $\sin^2 x + \cos^2 x = 1$

b). Untuk m dan n genap, tuliskan $\sin^m x$ dan $\cos^n x$ menjadi jumlah suku-suku dalam cosinus, gunakan identitas $\cos 2x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$

Contoh :

$$\begin{aligned}\int \sin^3 x \cos^2 x dx &= \int \sin^2 x \cos^2 x \sin x dx = -\int (1 - \cos^2 x) \cos^2 x d(\cos x) \\ &= -\int (\cos^2 x - \cos^4 x) d(\cos x) \\ &= \frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C\end{aligned}$$

$$\begin{aligned}
\int \sin^2 x \cos^2 x \, dx &= \int \frac{1 - \cos 2x}{2} \frac{1 + \cos 2x}{2} \, dx \\
&= \frac{1}{4} \int (1 + \cos^2 2x) \, dx = \frac{1}{4} \left(\int 1 + \frac{1 + \cos 4x}{2} \, dx \right) \\
&= \frac{3}{8} \int dx + \frac{1}{8} \int \cos 4x \, dx \\
&= \frac{3}{8} x + \frac{1}{32} \sin 4x + C
\end{aligned}$$

Bentuk $\int \tan^m x \sec^n x dx$ dan $\int \cot^m x \csc^n x dx$

Gunakan identitas

$$\tan^2 x = \sec^2 x - 1, \cot^2 x = \csc^2 x - 1$$

serta turunan tangen dan kotangen

$$d(\tan x) = \sec^2 x dx, d(\cot x) = -\csc^2 x dx$$

Contoh

$$\begin{aligned} \text{a. } \int \tan^4 x dx &= \int \tan^2 x \tan^2 x dx = \int \tan^2 x (\sec^2 x - 1) dx \\ &= \int \tan^2 x \sec^2 x dx - \int \tan^2 x dx \\ &= \int \tan^2 x d(\tan x) - \int (\sec^2 x - 1) dx \\ &= \frac{1}{3} \tan^3 x - \tan x + x + C \end{aligned}$$

$$\begin{aligned} \text{b. } \int \tan^2 x \sec^4 x \, dx &= \int \tan^2 x \sec^2 x \sec^2 x \, dx \\ &= \int \tan^2 x (1 + \tan^2 x) d(\tan x) \\ &= \int \tan^2 x + \tan^4 x \, dx \\ &= \frac{1}{5} \tan^5 x + \frac{1}{3} \tan^3 x + C \end{aligned}$$

Soal Latihan

Hitung

1. $\int \sin^4 x \cos^5 x dx$

2. $\int_0^{\pi/4} \tan^4 t \sec^2 t dt$

3. $\int \sec^4 x dx$

4. $\int \cot^2 w \csc^4 w dw$

5. $\int \csc^3 x dx$

9.3 Substitusi Trigonometri

a. Integralan memuat bentuk $\sqrt{a^2 - x^2}$, misal $x = a \sin t$

Contoh Hitung $\int \frac{\sqrt{25 - x^2}}{x^2} dx$

$$\int \frac{\sqrt{25 - x^2}}{x^2} dx = \int \frac{\sqrt{25 - 25 \sin^2 t} \cdot 5 \cos t dt}{25 \sin^2 t}$$

$$= \int \frac{\sqrt{25(1 - \sin^2 t)}}{5 \sin^2 t} \cos t dt = \int \frac{\cos^2 t}{\sin^2 t} dt = \int \cot^2 t dt$$

$$= \int (\csc^2 t - 1) dt = -\cot t - t + C$$

$$= -\frac{\sqrt{25 - x^2}}{x} - \sin^{-1}\left(\frac{x}{5}\right) + C$$

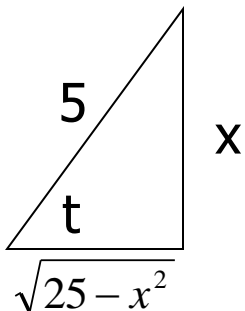
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Misal $x = 5 \sin t$

$dx = 5 \cos t dt$



b. Integralan memuat bentuk $\sqrt{a^2 + x^2}$, misal $x = a \tan t$

Contoh Hitung $\int \frac{1}{x^2 \sqrt{25 + x^2}} dx$

$$\int \frac{1}{x^2 \sqrt{25 + x^2}} dx = \int \frac{5 \sec^2 t dt}{25 \tan^2 t \sqrt{25 + 25 \tan^2 t}}$$

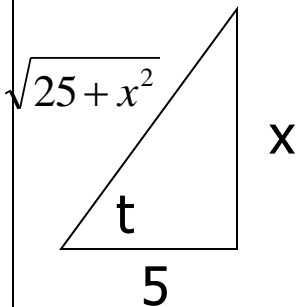
$$= \frac{1}{25} \int \frac{\sec^2 t dt}{\tan^2 t \sec t} = \frac{1}{25} \int \frac{\cos t}{\sin^2 t} dt = \frac{1}{25} \int \frac{d(\sin(t))}{\sin^2 t}$$

$$= -\frac{1}{25 \sin t} + C = -\frac{\sqrt{25 + x^2}}{25x} + C$$

Misal $x = 5 \tan t$

$$dx = 5 \sec^2 t dt$$

$$\tan t = \frac{x}{5}$$



c. Integralan memuat bentuk $\sqrt{x^2 - a^2}$, misal $x = a \sec t$

Contoh Hitung $\int \frac{1}{x^2 \sqrt{x^2 - 25}} dx$

$$\int \frac{1}{x^2 \sqrt{x^2 - 25}} dx = \int \frac{5 \sec t \tan t dt}{25 \sec^2 t \sqrt{25 \sec^2 t - 25}}$$

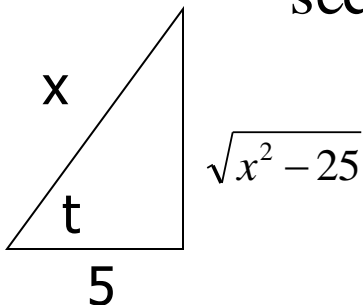
$$= \frac{1}{25} \int \frac{\sec t \tan t dt}{\sec^2 t \tan t} = \frac{1}{25} \int \frac{\sec t}{\sec^2 t} dt = \frac{1}{25} \int \cos t dt$$

$$= \frac{1}{25} \sin t + C = \frac{\sqrt{x^2 - 25}}{25x} + C$$

Misal $x = 5 \sec t$

$$dx = 5 \sec t \tan t dt$$

$$\sec t = \frac{x}{5}$$



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Soal Latihan

Hitung

1. $\int \frac{x^2 dx}{\sqrt{9-x^2}}$

2. $\int \frac{2x-3}{\sqrt{4-x^2}} dx$

3. $\int \frac{dx}{x^2 \sqrt{4-x^2}}$

4. $\int \frac{dx}{x \sqrt{x^2+9}}$

5. $\int \frac{dx}{x^2 \sqrt{x^2-16}}$

6. $\int \frac{dx}{(x^2+9)^{3/2}}$

7. $\int \frac{3x dx}{\sqrt{x^2+2x+5}}$

8. $\int \sqrt{5-4x-x^2} dx$

9. $\int \frac{2x+1}{x^2+2x+2} dx$

Substitusi Bentuk Akar

Integran memuat $\sqrt[n]{ax + b}$, misal $u = \sqrt[n]{ax + b}$

Contoh Hitung $\int \frac{dx}{2 + 2\sqrt{x}}$

Jawab :

$$\int \frac{dx}{2 + 2\sqrt{x}} = \int \frac{2udu}{2 + 2u} = \int \frac{u}{u + 1} du$$

$$= \int \frac{u + 1 - 1}{u + 1} du = \int \left(1 - \frac{1}{u + 1}\right) du$$

$$= u - \ln(u + 1) + C$$

$$= \sqrt{x} - \ln(1 + \sqrt{x}) + C$$

Misal $u = \sqrt{x} \rightarrow u^2 = x$

Dengan turunan implisit

$$2u \frac{du}{dx} = 1 \rightarrow dx = 2u du$$

Soal Latihan

Hitung

1. $\int x \sqrt[3]{x+4} \, dx$

2. $\int \frac{x^2 + 2x}{\sqrt{x+1}} \, dx$

3. $\int \frac{\sqrt{t}}{t+1} \, dt$

4. $\int x\sqrt{x+1} \, dx$

5. $\int \frac{t}{\sqrt{3t+4}} \, dt$

6. $\int x(1-x)^{2/3} \, dx$

9.4 Integral Fungsi Rasional

- Integran berbentuk fungsi rasional : $f(x) = \frac{P(x)}{Q(x)}$, der (P) < der(Q)
- Ada 4 kasus dari pemfaktoran penyebut (Q(x)) yaitu :
 1. Faktor linear tidak berulang.
 2. Faktor linear berulang.
 3. Faktor kuadratik tidak berulang.
 4. Faktor kuadratik berulang.

- **Kasus 1 (linier tidak berulang)**

Misal $Q(x) = (a_1 x + b_1)(a_2 x + b_2) \dots (a_n x + b_n)$

maka,
$$\frac{P(x)}{Q(x)} \equiv \frac{A_1}{a_1 x + b_1} + \frac{A_2}{a_2 x + b_2} + \dots + \frac{A_n}{a_n x + b_n}$$

dengan A_1, A_2, \dots, A_n konstanta yang dicari.

Contoh Hitung $\int \frac{x+1}{x^2-9} dx$

Jawab

Faktorkan penyebut : $x^2 - 9 = (x - 3)(x + 3)$

$$\frac{x+1}{x^2-9} = \frac{A}{(x+3)} + \frac{B}{(x-3)} = \frac{A(x-3) + B(x+3)}{(x-3)(x+3)}$$

$$\Leftrightarrow x+1 = A(x-3) + B(x+3) = (A+B)x + (-3A+3B)$$

Samakan koefisien ruas kiri dan ruas kanan

$$\begin{array}{l|l} A + B = 1 & \times 3 \\ -3A + 3B = 1 & \times 1 \end{array} \quad \longrightarrow \quad \begin{array}{l} 3A + 3B = 3 \\ -3A + 3B = 1 \end{array}$$

$$\frac{6B = 4}{6B = 4} \quad \longrightarrow \quad B = 2/3, A = 1/3$$

Sehingga

$$\int \frac{x+1}{x^2-9} dx = \int \frac{1/3}{(x+3)} dx + \int \frac{2/3}{(x-3)} dx = \frac{1}{3} \ln |x+3| + \frac{2}{3} \ln |x-3| + C$$

Kasus 2 Linear berulang

Misal $Q(x) = (a_i x + b_i)^p$

Maka

$$\frac{P(x)}{Q(x)} \equiv \frac{A_1}{(a_i x + b_i)} + \frac{A_2}{(a_i x + b_i)^2} + \dots + \frac{A_{p-1}}{(a_i x + b_i)^{p-1}} + \frac{A_p}{(a_i x + b_i)^p}$$

dengan konstanta $A_1, A_2, \dots, A_{p-1}, A_p$ akan dicari

Contoh Hitung $\int \frac{1}{(x+2)^2(x-1)} dx$

Jawab

$$\frac{1}{(x+2)^2(x-1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1}$$

$$\frac{1}{(x+2)^2(x-1)} = \frac{A(x+2)(x-1) + B(x-1) + C(x+2)^2}{(x+2)^2(x-1)}$$

$$1 = A(x+2)(x-1) + B(x-1) + C(x+2)^2$$

$$1 = (A+C)x^2 + (A+B+4C)x + (4C-2A-B)$$

Penyebut ruas kiri =
penyebut ruas kanan

$$A+C=0$$

$$A+B+4C=0$$

$$-2A-B+4C=1$$

$$A+B+4C=0$$

$$-2A-B+4C=1$$

$$-A+8C=1$$

$$A+C=0$$

$$-A+8C=1$$

$$9C=1$$

$$B=-1/3$$

$$A=-1/9$$

$$C=1/9$$

$$\int \frac{1}{(x+2)^2(x-1)} dx = \frac{-1}{9} \int \frac{1}{(x+2)} dx - \frac{1}{3} \int \frac{1}{(x+2)^2} dx + \frac{1}{9} \int \frac{1}{(x-1)} dx$$

$$= -\frac{1}{9} \ln |x+2| + \frac{1}{3(x+2)} + \frac{1}{9} \ln |x-1| + C$$

Kasus 3 Kuadrat tak berulang

Misal

$$Q(x) = (a_1 x^2 + b_1 x + c_1)(a_2 x^2 + b_2 x + c_2) \dots (a_n x^2 + b_n x + c_n)$$

Maka

$$\frac{P(x)}{Q(x)} \equiv \frac{A_1 x + B_1}{a_1 x^2 + b_1 x + c_1} + \frac{A_2 x + B_2}{a_2 x^2 + b_2 x + c_2} + \dots + \frac{A_n x + B_n}{a_n x^2 + b_n x + c_n}$$

Dengan A_1, A_2, \dots, A_n , dan B_1, B_2, \dots, B_n konstanta yang akan dicari

Contoh Hitung $\int \frac{dx}{x(x^2 + 1)}$

Jawab

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1) + (Bx+C)x}{x(x^2+1)}$$

$$1 = A(x^2+1) + (Bx+C)x \longrightarrow 1 = (A+B)x^2 + cx + A$$

$$A+B=0$$

$$C=0$$

$$A=1$$

$$\longrightarrow B=-1$$

$$\int \frac{x}{(x^2+1)} dx = \int \frac{x}{x^2+1} \frac{d(x^2+1)}{2x}$$
$$= \frac{1}{2} \int \frac{d(x^2+1)}{x^2+1}$$

$$\int \frac{1}{x(x^2+1)} dx = \int \frac{1}{x} dx - \int \frac{x}{(x^2+1)} dx$$
$$= \ln |x| - \frac{1}{2} \ln(x^2+1) + C$$

Kasus 4 Kuadratik berulang

Misal $Q(x) = (a_i x^2 + b_i x + c_i)^p$

Maka

$$\frac{P(x)}{Q(x)} \equiv \frac{A_1 x + B_1}{(a_i x^2 + b_i x + c_i)} + \frac{A_2 x + B_2}{(a_i x^2 + b_i x + c_i)^2} + \dots + \frac{A_{p-1} x + B_{p-1}}{(a_i x^2 + b_i x + c_i)^{p-1}} + \frac{A_p x + B_p}{(a_i x^2 + b_i x + c_i)^p}$$

Dimana $A_1, A_2, \dots, A_{p-1}, A_p$ dan $B_1, B_2, \dots, B_{p-1}, B_p$ konstanta yang akan dicari

Contoh Hitung $\int \frac{6x^2 - 15x + 22}{(x+3)(x^2+2)^2} dx$

Jawab :

$$\frac{6x^2 - 15x + 22}{(x+3)(x^2+2)^2} = \frac{A}{x+3} + \frac{Bx+C}{x^2+2} + \frac{Dx+E}{(x^2+2)^2}$$
$$= \frac{A(x^2+2)^2 + (Bx+C)(x^2+2)(x+3) + (Dx+E)(x+3)}{(x+3)(x^2+2)^2}$$

$$6x^2 - 15x + 22 = A(x^2+2)^2 + (Bx+C)(x^2+2)(x+3) + (Dx+E)(x+3)$$

$$6x^2 - 15x + 22 = (A+B)x^4 + (3B+C)x^3 + (4A+2B+3C+D)x^2 + (6B+2C+3D+E)x + (4A+6C+3E)$$

Dengan menyamakan koefisien ruas kiri dan kanan diperoleh

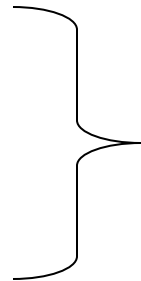
$$A+B=0$$

$$3B+C=0$$

$$4A+2B+3C+D=1$$

$$6B+2C+3D+E=-15$$

$$4A+6C+3E=22$$



Dengan eliminasi : $A=1, B=-1, C=3$
 $D=-5, E=0$

Sehingga

$$\begin{aligned} \int \frac{6x^2-15x+22}{(x+3)(x^2+2)^2} dx &= \int \frac{1}{(x+3)} dx - \int \frac{x-3}{(x^2+2)} dx - 5 \int \frac{x}{(x^2+2)^2} dx \\ &= \int \frac{dx}{x+3} - \frac{1}{2} \int \frac{2x}{x^2+2} dx + 3 \int \frac{dx}{x^2+2} - \frac{5}{2} \int \frac{2x}{(x^2+2)^2} dx \\ &= \ln |x+3| - \frac{1}{2} \ln(x^2+2) + \frac{3}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{5}{2(x^2+2)} + C. \end{aligned}$$

Catatan jika $der(P(x)) \geq der(Q(x))$, bagi terlebih dahulu P(x) dengan Q(x), sehingga

$$\frac{P(x)}{Q(x)} = H(x) + \frac{S(x)}{Q(x)}, \quad der(S(x)) < der(Q(x))$$

Contoh Hitung

$$\int \frac{x^3 + 2x^2 + x - 4}{x^2 - 4} dx \quad \longrightarrow \quad Der(P(x))=3 > der(Q(x))=2$$

Bagi terlebih dahulu P(x) dengan Q(x)

$$\begin{array}{r}
 \overline{) x^3 + 2x^2 + x - 4} \\
 \underline{x^3 - 4x} \\
 2x^2 + 5x - 4 \\
 \underline{2x^2 - 8} \\
 5x + 4
 \end{array}
 \quad \longrightarrow \quad
 \frac{x^3 + 2x^2 + x - 4}{x^2 - 4} = x + 2 + \frac{5x + 4}{x^2 - 4}$$

$$\frac{5x+4}{x^2-4} = \frac{5x+4}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$= \frac{A(x+2) + B(x-2)}{(x-2)(x+2)}$$

$$5x+4 = A(x+2) + B(x-2) \dots\dots\dots(*)$$

Persamaan (*) berlaku untuk sembarang x, sehingga berlaku juga untuk
 Untuk x=2 dan x=-2

$$\text{Untuk } x = 2 \longrightarrow 5 \cdot 2 + 4 = A(2+2) \longrightarrow A = 7/2$$

$$\text{Untuk } x = -2 \longrightarrow 5 \cdot (-2) + 4 = B(-2-2) \longrightarrow B = 3/2$$

Dengan menggunakan hasil diatas :

$$\int \frac{x^3 + 2x^2 + x - 4}{x^2 - 4} dx = \int (x+2) dx + \frac{7}{2} \int \frac{1}{x-2} dx + \frac{3}{2} \int \frac{1}{x+2} dx$$

$$= \frac{1}{2} x^2 + 2x + \frac{7}{2} \ln |x-2| + \frac{3}{2} \ln |x+2| + C$$

Soal Latihan

Hitung

$$1. \int \frac{2x-1}{x^2-6x+18} dx$$

$$2. \int \frac{1}{(x+5)^2(x-1)} dx$$

$$3. \int \frac{5x^2+3x-2}{x^3+2x^2} dx$$

$$4. \int \frac{dx}{x(x^2+1)^2}$$

$$5. \int_2^5 \frac{x^2+2x}{x^3+3x^2+4} dx$$

$$6. \int \frac{2x^2-3x-36}{(2x-1)(x^2+9)} dx$$

$$7. \int \frac{x^3+x^2}{x^2+5x+6} dx$$

Integral Fungsi Rasional dalam sin dan cos

$$\int f(\cos x, \sin x) dx = ? , f \text{ fungsi rasional}$$

Cara :

Gunakan substitusi $\tan \frac{x}{2} = t$, dari sini dapat diperoleh

$$\frac{dt}{dx} = \sec^2\left(\frac{x}{2}\right) \frac{1}{2} \quad \Rightarrow \quad dx = \frac{2}{1 + \tan^2\left(\frac{x}{2}\right)} dt \quad \Rightarrow \quad dx = \frac{2}{1 + t^2} dt$$

$$\begin{aligned} \sin x &= 2 \sin \frac{1}{2} x \quad \cos \frac{1}{2} x = 2 \frac{\sin \frac{1}{2} x}{\cos \frac{1}{2} x} \cos^2 \frac{x}{2} \\ &= 2 \frac{\tan \frac{x}{2}}{\sec^2 \frac{x}{2}} = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{2t}{1 + t^2} \end{aligned}$$

$$\begin{aligned}\cos x &= 2 \cos^2 \frac{x}{2} - 1 = \frac{2}{\sec^2 \frac{x}{2}} - 1 = \frac{2}{1 + \tan^2 \frac{x}{2}} - 1 = \frac{2}{1 + t^2} - 1 \\ &= \frac{1 - t^2}{1 + t^2}\end{aligned}$$

Contoh Hitung

$$\int \frac{dx}{1 + \sin x + \cos x}$$

Jawab

Gunakan substitusi diatas diperoleh

$$\frac{1}{1 + \sin x + \cos x} = \frac{1}{1 + \frac{1 - t^2}{1 + t^2} + \frac{2t}{1 + t^2}} = \frac{1 + t^2}{1 + t^2 + 1 - t^2 + 2t} = \frac{1 + t^2}{2(1 + t)}$$

$$\int \frac{dx}{1 + \sin x + \cos x} = \int \frac{1 + t^2}{2(1 + t)} \frac{2}{1 + t^2} dt = \int \frac{1}{1 + t} dt = \ln |1 + t| + C = \ln |1 + \tan \frac{x}{2}| + C$$

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Soal Latihan

Hitung

1. $\int \frac{dx}{1 - \sin x + \cos x}$

2. $\int \frac{dx}{3 + 5 \sin x}$

3. $\int \frac{\cos x}{1 + \cos x} dx$

4. $\int \frac{dx}{\sin x + \tan x}$

5. $\int \frac{\cot x}{1 + \sin x}$