

MATERI :

- Diferensial fungsi dua variable orde dua

Diferensial parsial orde dua Fungsi Dua variable

$$\text{Bentuk } z = f(x,y)$$

Diferensial parsial orde dua satu Fungsi Dua variable bentuk $z = f(x,y)$ ada tiga macam yaitu:

$$1. \frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial x} \right)$$

$$2. \frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial y} \right)$$

$$3. \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x} \right) = \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right)$$

Contoh-contoh :

Tentukan $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y^2}$ serta $\frac{\partial^2 z}{\partial x \partial y}$ dari fungsi dua variable berikut :

1. $z = x^3 + y^3 + 4xy$

Jawab:

$$\frac{\partial z}{\partial x} = 3x^2 + 0 + 4y$$

$$\frac{\partial^2 z}{\partial x^2} = 6x + 0 + 0$$

$$\frac{\partial z}{\partial y} = 0 + 3y^2 + 4x$$

$$\frac{\partial^2 z}{\partial y^2} = 6y$$

$$\frac{\partial^2 z}{\partial x \partial y} = 4$$

2. $z = \ln (x^2 + y^2)$

Jawab :

$$\frac{\partial z}{\partial x} = \frac{1}{x^2 + y^2} \cdot (2x + 0)$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{(x^2 + y^2)2 - 2x(2x)}{(x^2 + y^2)^2}$$

$$\frac{\partial z}{\partial y} = \frac{1}{x^2 + y^2} \cdot (0 + 2y)$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{(x^2 + y^2)2 - 2y(2y)}{(x^2 + y^2)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{(x^2 + y^2)0 - 2y(2x)}{(x^2 + y^2)^2} = \frac{-4xy}{(x^2 + y^2)^2}$$

3. $z = e^{-4x+5y}$

Jawab :

$$\frac{\partial z}{\partial x} = e^{-4x+5y} \cdot (-4 + 0)$$

$$\frac{\partial^2 z}{\partial x^2} = e^{-4x+5y} \cdot (-4)(-4)$$

$$\frac{\partial z}{\partial y} = e^{-4x+5y} \cdot (0 + 5)$$

$$\frac{\partial^2 z}{\partial y^2} = e^{-4x+5y} \cdot (5)(5)$$

$$\frac{\partial^2 z}{\partial x \partial y} = e^{-4x+5y} \cdot (-4)(5)$$

$$4. x^3 + y^3 + 4xyz = 10$$

Jawab:

Turunan terhadap x :

$$3x^2 + 0 + 4yz + 4xy \frac{\partial z}{\partial x} = 0$$

$$4xy \frac{\partial z}{\partial x} = -3x^2 - 4yz$$

$$\frac{\partial z}{\partial x} = \frac{-3x^2 - 4yz}{4xy}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{4xy(-6x - 4y \frac{\partial z}{\partial x}) - (-3x^2 - 4yz)(4y)}{(4xy)^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{4xy(-6x - 4y(\frac{-3x^2 - 4yz}{4xy})) - (-3x^2 - 4yz)(4y)}{(4xy)^2}$$

Turunan terhadap y :

$$0 + 3y^2 + 4xz + 4xy \frac{\partial z}{\partial y} = 0$$

$$4xy \frac{\partial z}{\partial y} = -3y^2 - 4xz$$

$$\frac{\partial z}{\partial y} = \frac{-3y^2 - 4xz}{4xy}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{4xy(-6y - 4x \frac{\partial z}{\partial y}) - (-3y^2 - 4xz)(4x)}{(4xy)^2}$$

$$= \frac{4xy(-6y - 4y(\frac{-3y^2 - 4xz}{4xy})) - (-3y^2 - 4xz)(4x)}{(4xy)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{4xy(-0 - 4z - 4x \frac{\partial z}{\partial x}) - (-3x^2 - 4xz)(4y)}{(4xy)^2}$$

5). $x^3 + y^3 + z^3 = 10z$

Jawab:

Turunan terhadap x :

$$3x^2 + 0 + 3z^2 \frac{\partial z}{\partial x} = 10 \frac{\partial z}{\partial x}$$

$$3z^2 \frac{\partial z}{\partial x} - 10 \frac{\partial z}{\partial x} = -3x^2$$

$$(3z^2 - 10) \frac{\partial z}{\partial x} = -3x^2$$

$$\frac{\partial z}{\partial x} = \frac{-3x^2}{(3z^2 - 10)}$$

$$\begin{aligned} \frac{\partial^2 z}{\partial x^2} &= \frac{(3z^2 - 10)(-6x) - (-3x^2)(6z \frac{\partial z}{\partial x})}{(3z^2 - 10)^2} \\ &= \frac{(3z^2 - 10)(-6x) - (-3x^2)(6z \frac{-3x^2}{(3z^2 - 10)})}{(3z^2 - 10)^2} \end{aligned}$$

Turunan terhadap y :

$$0 + 3y^2 + 3z^2 \frac{\partial z}{\partial y} = 10 \frac{\partial z}{\partial y}$$

$$3z^2 \frac{\partial z}{\partial y} - 10 \frac{\partial z}{\partial y} = -3y^2$$

$$(3z^2 - 10) \frac{\partial z}{\partial y} = -3y^2$$

$$\frac{\partial z}{\partial y} = \frac{-3y^2}{(3z^2 - 10)}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{(3z^2 - 10)(-6y) - (-3y^2)(6z \frac{\partial z}{\partial y})}{(3z^2 - 10)^2}$$

$$= \frac{(3z^2 - 10)(-6y) - (-3y^2)(6z \frac{-3y^2}{(3z^2 + 10)})}{(3z^2 - 10)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{(3z^2 - 10)(-0) - (-3y^2)(6z \frac{\partial z}{\partial x})}{(3z^2 - 10)^2}$$

$$= \frac{-(-3y^2)(6z \frac{-3x^2}{(3z^2 + 10)})}{(3z^2 - 10)^2}$$

6). $\ln(x^3 + y^3 + z^3) = 16$

Jawab:

Turunan terhadap x :

$$\frac{1}{x^3 + y^3 + z^3} (3x^2 + 0 + 3z^2 \frac{\partial z}{\partial x}) = 0$$

$$(3x^2 + 0 + 3z^2 \frac{\partial z}{\partial x}) = 0$$

$$3z^2 \frac{\partial z}{\partial x} = -3x^2$$

$$\frac{\partial z}{\partial x} = \frac{-3x^2}{3z^2} = \frac{-x^2}{z^2}$$

$$\begin{aligned} \frac{\partial^2 z}{\partial x^2} &= \frac{z^2(-2x) - (-x^2)(2z \frac{\partial z}{\partial x})}{(z^2)^2} \\ &= \frac{z^2(-2x) - (-x^2)(2z \frac{-x^2}{z^2})}{(z^2)^2} \end{aligned}$$

Turunan terhadap y :

$$\frac{1}{x^3 + y^3 + z^3} (0 + 3y^2 + 3z^2 \frac{\partial z}{\partial y}) = 0$$

$$(0 + 3y^2 + 3z^2 \frac{\partial z}{\partial y}) = 0$$

$$3z^2 \frac{\partial z}{\partial y} = -3y^2$$

$$\frac{\partial z}{\partial y} = \frac{-3y^2}{3z^2} = \frac{-y^2}{z^2}$$

$$\begin{aligned} \frac{\partial^2 z}{\partial y^2} &= \frac{z^2(-2y) - (-y^2)(2z \frac{\partial z}{\partial y})}{(z^2)^2} \\ &= \frac{z^2(-2y) - (-y^2)(2z \frac{-y^2}{z^2})}{(z^2)^2} \end{aligned}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{z^2(0) - (-y^2)(2z \frac{\partial z}{\partial x})}{(z^2)^2} = \frac{-(-y^2)(2z \frac{-x^2}{z^2})}{(z^2)^2}$$

TUGAS:

Tentukan $\frac{\partial^2 z}{\partial x^2}$ dan $\frac{\partial^2 z}{\partial y^2}$ serta $\frac{\partial^2 z}{\partial x \partial y}$ dari fungsi dua variable berikut :

1. $z = \ln(3x^2 + 6y^3)$

2. $z = \cos (4xy + 7 x^5)$

3. $z = e^{\sin(3x+7y)}$

4. $xy^2z + \ln(4x^3) - \sec 7y = 6z$

5. $5xyz + e^{9z} + 5z^2 = \sin 9y$

LINK INTERNAL

LINK EKSTERNAL

LINK DOKUMEN :

Murray R. Spigel JR, KALKULUS LANJUTAN, , Erlangga , Jakarta
1991